The Temperature Paradox and Russell's Analysis of the Definite Determiner*

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Abstract: Lasersohn has argued that the use of Russell's analysis of the definite determiner in Montague Grammar, which is responsible for giving the correct prediction in the case of the Temperature Paradox, is also responsible for giving the wrong prediction in the case of the Gupta Syllogism. In this paper I argue against Lasersohn, and show that the problem with the Gupta Syllogism can be solved by making a minor addition to Montague Grammar. This solution is one that Lasersohn discusses but rejects. I will show that his critique of it is ill-founded.

Keywords: Montague Grammar, Intensionality, Temperature Paradox, Gupta Syllogism, Definite Determiner, Meaning Postulates

1 Introduction

The argument

The temperature is ninety The temperature rises Ninety rises

is intuitively invalid, but a naïve formalization of it in some extensional logic will give something like

$$\exists y (\forall x (\mathbf{temp}(x) \leftrightarrow x = y) \land y = \mathbf{90}) \\ \exists y (\forall x (\mathbf{temp}(x) \leftrightarrow x = y) \land \mathbf{rise}(y)) \\ \mathbf{rise}(\mathbf{90})$$

which is valid (given any reasonable semantics). This is the so-called temperature paradox (also know as "Partee's Paradox"). Montague (1973) discusses the paradox and shows that the argument is correctly translated as an invalid argument in his system. But Lasersohn (2005) shows that this virtue of Montague's system can be turned into a vice: the argument

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Necessarily, the temperature is the price
The temperature rises
The price rises

(known as the Gupta Syllogism¹) is intuitively *valid* but is also predicted to be invalid by Montague Grammar. Lasersohn argues that the problem is due to the use of Russell's analysis of the definite determiner in Montague Grammar, and proposes a replacement based on Frege's presuppositional analyses.

According to Russell, the sentence *The temperature rises* is true if there is a unique temperature and this temperature has the property of rising, and false otherwise. According to Frege, the sentence is true if there is a unique temperature and this temperature has the property of rising; false if there is a unique temperature and this temperature does not have the property of rising; and undefined if there is no temperature or it is not unique. In other words, Russell and Frege agree on the truth condition of the sentence, but where Russell's analysis results in a bivalent semantics, Frege claims that the sentence lacks a truth value if *the temperature* lacks a referent.

In this paper I will argue against Lasersohn and show that the problem of the Gupta Syllogism can be solved by making a minor addition to Montague Grammar. This solution is one that Lasersohn discusses but rejects. I will show that his critique of it is ill-founded.

In addition to this introduction and the conclusion, this paper contains three sections. The first concerns Montague's solution to the temperature paradox. Laser-sohn's case is laid out in the second. In the third I argue against Lasersohn, and show that what I will call the interpretation-restriction solution does not suffer from the alleged problems.

2 Montague's Solution to the Temperature Paradox

The reason that the first argument mentioned above is intuitively invalid is that the two occurrences of the word *temperature* do not have the same referent. In the second premise the word refers to the function from instants of time to the temperature at that instant, while in the first premise the word just refers to the value of that function for the present instant of time. The translation of the argument in Montague Grammar² is in sync with these intuitions:

the temperature is ninety	\mapsto	$\exists y (\forall x (\mathbf{temp}_t(x) \leftrightarrow x = y) \land y_t = 90_t)$
the temperature rises	\mapsto	$\exists y (\forall x (\mathbf{temp}_t(x) \leftrightarrow x = y) \land \mathbf{rise}_t(y))$
ninety rises	\mapsto	$\mathbf{rise}_t(\lambda t. 90_t)$

¹At least that is what it will be known as in this paper. To be more precise, it is a variant of the Gupta Syllogism. It first appeared in the literature in Dowty, Wall, and Peters 1981.

 $^{^{2}}$ I will use a Ty2 version of Montague Grammar. The indices will consist of just instants of time, as it will not be necessary to consider more than one possible world. Functional application to a time index will be written with the index as subscript.

			temp	\mathbf{rise}	price
$i_0 \mapsto \begin{cases} \begin{bmatrix} i_0 \mapsto 90\\ i_1 \mapsto 90\\ \\ i_0 \mapsto 90\\ \\ i_1 \mapsto 100\\ \\ i_0 \mapsto 100\\ \\ i_1 \mapsto 90\\ \\ i_0 \mapsto 100\\ \\ i_1 \mapsto 100 \end{cases}$	$\left[\begin{array}{c}i_0\mapsto 90\\i_1\mapsto 90\end{array}\right]$	\mapsto	0	0	1
	$\left[\begin{array}{c}i_0\mapsto 90\\i_1\mapsto 100\end{array}\right]$	\mapsto	1	1	0
	$\left[\begin{array}{c}i_0\mapsto 100\\i_1\mapsto 90\end{array}\right]$	\mapsto	0	0	0
	$\left[\begin{array}{c}i_0\mapsto 100\\i_1\mapsto 100\end{array}\right]$	\mapsto	0	0	0
$i_1 \mapsto \left\{ \begin{array}{c} \\ \end{array} \right\}$	$\left[\begin{array}{c}i_0\mapsto 90\\i_1\mapsto 90\end{array}\right]$	\mapsto	0	0	0
	$\left[\begin{array}{c}i_0\mapsto 90\\i_1\mapsto 100\end{array}\right]$	\mapsto	0	1	1
	$\left[\begin{array}{c}i_0\mapsto 100\\i_1\mapsto 90\end{array}\right]$	\mapsto	0	0	0
	$\left[\begin{array}{c}i_0\mapsto 100\\i_1\mapsto 100\end{array}\right]$	\mapsto	1	0	0

Figure 1: A model

Here, y is a variable of type (s, e), so the property of rising is, in the second premise, attributed to a function from the set of instances of time to entities. However, it is just one value of this function that is equated with 90 in the first premise. And in the conclusion, the property of rising is attributed to the constant function to 90. This is due to Montague's first meaning postulate:

MP1: $\exists \nu \forall t (\nu = \mathbf{90}_t)$

From this it can be seen that the argument is invalid. For later comparison, let us specify a counter-model. Let the domain be just the two-element set $\mathcal{E} = \{90, 100\}$, and the set of instances of time also be a two-element set, namely $\mathcal{I} = \{i_0, i_1\}$. Obviously, we interpret **90** as the constant function from instances of time to 90. The constants temp and rise are of type (s, ((s, e), t)), so temp and rise must each be interpreted as an element of $(\{0, 1\}^{(\{90, 100\}^{\{i_0, i_1\}})})^{\{i_0, i_1\}}$. We define them as shown in figure 1. The figure is to be read as follows: temp is the function that sends, for example, i_0 to a certain function defined on a four-element set. One element of this set is the function $\{(i_0, 90), (i_1, 90)\}$ and it is mapped to 0. The figure also defines the interpretation of the constant price for use below.

This model satisfies MP1, and the two premises are true at i_0 while the conclusion is false at i_0 . To see this, it suffices to look at the top half of figure 1. The first premise is true because at i_0 there is a unique temperature function and it maps i_0 to 90. The second premise is true because at i_0 , according to the model, that function

is rising. And the conclusion is false because $\lambda t.90$ denotes the function that maps both i_0 and i_1 to 90, and at i_0 that function is not rising according to the model.

3 Lasersohn and the Gupta Syllogism

Now we turn to the Gupta Syllogism, which is translated as follows:

necessarily, the temperature is the price $\mapsto \quad \forall t (\exists y (\forall x (\mathbf{temp}_t(x) \leftrightarrow x = y) \land \exists v (\forall z (\mathbf{price}_t(z) \leftrightarrow z = v) \land y_t = v_t)))$ the temperature rises $\mapsto \quad \exists y (\forall x (\mathbf{temp}_t(x) \leftrightarrow x = y) \land \mathbf{rise}_t(y))$ the price rises $\mapsto \quad \exists y (\forall x (\mathbf{price}_t(x) \leftrightarrow x = y) \land \mathbf{rise}_t(y))$

The argument is intuitively valid,³ but formally invalid. The same model as above can be used as counter-model: again, the two premises are true and the conclusion false at i_0 . The first premise is true, in spite of the different interpretations of temp and price, because at both i_0 and i_1 there are unique temperature and price functions; at i_0 those functions map i_0 to the same value; and at i_1 they map i_1 to the same value. The second premise is true for the same reason as above. And the conclusion is false because the price function at i_0 , according to the model, is not rising.

3.1 Interpretation-Restriction Solution

One way to exclude counter-models is to introduce a meaning postulate restricting the possibility of modal variance of temp and price, such as

MP3: $\forall x \forall t(\delta_t(x) \rightarrow \forall t' \delta_{t'}(x))$, where $\delta \in \{\text{temp, price}\}$

This would rule out the specified model, among others. For it is not consistent with **MP3** for a function to be a temperature or a price at one instance of time but not at another. We will call this the "interpretation-restriction solution".

However, Lasersohn rejects this solution. His argument for doing so is as follows: One would think that if adding **MP3** were be a reasonable solution, it would be because the intuitive validity of the argument was due to the specific meanings of *temperature* and *price*. So it should be possible to replace these two words with other common nouns, such that the result is an intuitively invalid argument. But that is not the case; one sees for example that replacing *temperature* with *woman* and *price* with *fish* results in no significant change:

> Necessarily, the woman is the fish The woman rises The fish rises

³At least when *necessarily* is read as *necessarily always*, cf. Romero 2008.

So if we cling to this solution, we would, according to Lasersohn, have to extend **MP3** to hold for all common nouns, which does not seem reasonable: meaning postulates should be a device for restricting the set of logically possible models based on the specific meanings of a restricted class of words. So if there is an aspect of their meaning that all words of a grammatical class have in common, it should be reflected at a more fundamental level of the system.

On top of this, Lasersohn points to another problem that results from Montague's second meaning postulate. This meaning postulate has the effect that only constant functions from instances of time to entities can satisfy the predicate denoted by what Montague calls "ordinary" common nouns (as opposed to *temperature* and *price* which are "extraordinary" common nouns):

MP2: $\forall t(\delta_t(x) \to \exists \nu(x = \lambda t.\nu))$, where $\delta \in \{\text{woman, fish, philosopher}\}$

First a bit of terminology, taken from Schwager (2007): The type of the interpretations of the translations of the common nouns is (s, ((s, e), t)). We refer to the relativity of the interpretation that is due to the first s as the "outer index dependence", and to the relativity that is due to the the second s as the "inner index dependence". Given this terminology, the effect of **MP2** and **MP3** can be stated very simply: **MP2** neutralizes the inner index dependence and **MP3** neutralizes the outer index dependence. The second problem with the interpretation-restriction solution is that making both **MP2** and **MP3** hold for ordinary common nouns makes them completely independent of index, that is, completely modally invariant. So a philosopher would necessarily have to be a philosopher, and that is an unacceptable consequence.

For these reasons Lasersohn rejects the interpretation-restriction solution and proposes another, the "Fregean solution".

3.2 Fregean Solution

As temp is a constant of type (s, ((s, e), t)), it is interpreted as an element of $(\{0, 1\}^{(\mathcal{E}^{\mathcal{I}})})^{\mathcal{I}}$; that is, temperature is indexed for instants of time twice. As a consequence there are models (the one defined above being an example) in which the following holds: at i_0 it is the case that at i_0 the temperature is 90 degrees, but at i_1 it is the case that at i_0 the temperature is 100 degrees. Or to put it more simply: the truth about what the temperature is at a given moment may change over time – which makes no intuitive sense (except perhaps to the employees at the Ministry of Truth in George Orwell's 1984). This suggests that common nouns should be translated as constants of type (s, (e, t)) instead, so that they are interpreted as elements of $(\{0, 1\}^{\mathcal{E}})^{\mathcal{I}}$.

On the other hand, the translation of intransitive verbs as constants of type (s, ((s, e), t)) is quite reasonable. To see this, consider a model with four instants, i_0 , i_1 , i_2 and i_3 , and think of them as ordered by index (i.e., i_n is before i_m iff n < m). For the model to be intuitively reasonable, the extension of *The temperature rises* in i_1 should be 1 iff the temperature in i_0 is lower than in i_1 and that temperature

is again lower than in i_2 ; and the extension of *The temperature rises* in i_2 should be 1 iff the temperature in i_1 is lower than in i_2 and that temperature is again lower than in i_3 . This means that *rise* should be interpreted as something that, in different elements of \mathcal{I} , can take different sets of functions from \mathcal{I} to \mathcal{E} as its value. And that is exactly what the assignment of type (s, ((s, e), t)) makes possible.

So to solve the problem of the Gupta Syllogism, one should investigate possible ways of changing Montague's system such that common nouns are translated into constants of type (s, (e, t)) while intransitive verbs are not. The major challenge in doing the former without also doing the latter is the determiner *the* (and Lasersohn suggests that this was Montague's motivation for designing the system as he did). The term *the temperature* is translated as

$$\lambda X.\exists x (\forall y (\mathbf{temp}_t(y) \leftrightarrow x = y) \land X_t(x))$$

If the type of temp is changed to (s, (e, t)), the type of the variable y must be changed to e. But if rise is kept as a constant of type (s, ((s, e), t)), to which this λ -term can be applied, x's type must remain (s, e). So the change would render the subformula x = y ill-formed.

Lasersohn's solution is to replace the Russellian analysis of *the* with the presuppositional analysis of Frege. The specific changes Lasersohn proposes result in this alternative translation of the Gupta Syllogism:

necessarily, the temperature is the price	\mapsto	$\forall t(\nu \mathbf{temp}_t(\nu) = \nu \mathbf{price}_t(\nu))$
the temperature rises	\mapsto	$\mathbf{rise}_t(\lambda t. \nu \mathbf{temp}_t(\nu))$
the price rises	\mapsto	$\mathbf{rise}_t(\lambda t. \nu \mathbf{price}_t(\nu))$

If there is exactly one temperature/price, then $\nu \text{temp}_t(\nu)/\nu \text{price}_t(\nu))$ denotes it, and otherwise it is undefined. In the latter case, the sentence it is part of has no truth value.

Translated in this way, the argument is valid. To assist intuition, it can be helpful to consider a concrete example of how temp, price and rise can be interpreted in such a way that both premises and conclusion are true. Such an example is given in figure 2. Intuitively, the new model can be seen as the "reduction" of the old model as forced by the simpler type of common nouns.

The reason the argument is now valid is as follows: For the first premise to be true, it must be the case at every instant in the given model that there is a unique temperature and a unique price and that they are identical. That is, there is a function from instants to the temperature at that instant and a function from instants to the price at that instant, and they are identical. Ergo, the extension of $\lambda t. \nu \text{temp}_t(\nu)$ is identical to the extension of $\lambda t. \nu \text{price}_t(\nu)$ (and independent of index of evaluation). So if the second premise is true at some instant, the conclusion is also true at that instant.

It should be noted that the temperature paradox-argument is still invalid (i_0 being a counter-example). Further, there are still models in which $rise_t(\lambda t. \nu temp_t(\nu))$ has different truth values at different indices, as it has been argued above that there should be, even though $\lambda t. \nu temp_t(\nu)$ has the same extension in all elements of \mathcal{I} .

rise

Figure 2: A "reduction" of the model of figure 1

All this speaks in favour of Lasersohn's solution. We will now turn to the critique of it.

4 Discussion

4.1 Fregean Solution

Taking a cue from Romero (2008) we can use the following argument to see that the Fregean solution is flawed:

Necessarily, every temperature is a price A temperature rises A price rises

This is a minor modification of the Gupta Syllogism and is also intuitively valid. The translation (in Montague's system without the changes considered above) is as follows:

necessarily, every temperature is a price

 $\begin{array}{rcl} & \mapsto & \forall t (\forall x (\mathbf{temp}_t(x) \to \exists y (\mathbf{price}_t(y) \land x_t = y_t))) \\ a \ temperature \ rises & \mapsto & \exists x (\mathbf{temp}_t(x) \land \mathbf{rise}_t(x)) \\ a \ price \ rises & \mapsto & \exists x (\mathbf{price}_t(x) \land \mathbf{rise}_t(x)) \end{array}$

			temp
;	90	\mapsto	1
$i_0 \mapsto \left\{ \right\}$	100	\mapsto	1
$i_1 \mapsto \left\{ \right.$	90	\mapsto	1
	100	\mapsto	1

Figure 3: An unsuccessful modeling of a situation with two temperatures

The formal argument is invalid; once again the model shown in figure 1 is a counter-model (for essentially the same reasons that made it a counter-model to the Gupta Syllogism). And the solution proposed by Lasersohn cannot be extended to deal with this example. Why? Firstly, because Lasersohn's solution to the problem with the Gupta Syllogism was dependent on the existence of an alternative analysis of *the* in "the marked" than the one built into Montague's system. But the formalizations of *a* and *every* are uncontroversial, so it is very difficult to see how they could be changed in an intuitively acceptable way to fit with temp and price being of type (s, (e, t)).

However, the phrase "it is very difficult to see" normally signals that an argument is less than completely convincing, and this is no exception; it could simply be a lack of creativity that prevents us from seeing a good way to change the semantics of *a* and *every*. Hence, a better argument is needed. So, secondly, recall that a temperature is (in one sense of the word) intuitively a function from instants of time to numbers. Such functions are included as "parts" of the interpretation of temp in Montague's system (the contents of the square brackets in figure 1), but they are not in a direct sense parts of the interpretation of temp in Lasersohn's modification of this system. In the latter, there are only individual temperatures (i.e., numbers) at each instant, and then the temperature function is recovered from these, as the extension of $\lambda t.nv temp_t(v)$. But that only works when the temperature function is unique. Consider a situation in which there are two temperature functions (say, the temperature in two different cities), the first rising from 90 to 100 between today and tomorrow and the second falling from 100 to 90. With the interpretation of temp being an element of $(\{0, 1\}^{\mathcal{E}})^{\mathcal{I}}$, this would have to be modeled as in figure 3.

The problem is that the two functions cannot be recovered from this, for it is indistinguishable from a situation in which the first temperature function is constant at 90 and the second constant at 100 - to say nothing of situations with more than two temperature functions.

Schwager (2007) tries to solve this problem while adhering to Lasersohn's strategy. Her idea is, basically, that temperatures are in a sense always unique. For a temperature is always the temperature of something, and unique as such. For instance, the sentence *Every temperature is rising* should be read as "Every object xis such that the temperature of x is rising". Building on this idea, every sentence containing *temperature* can be rephrased in such a way that every occurrence of this word is preceded by *the*, thus eliminating the problem associated with Lasersohn's solution.

Schwager's solution is rather complex and involves, among other things, the use of different types for ordinary and extraordinary common nouns, as well as two different translations of each quantificational determiner (such as *every* and *most*). It will lead too far to go into this solution here. Instead, I will show that another solution – which, if nothing else, is simpler and probably more loyal to the spirit of Montague Grammar – can solve the problem: the interpretation-restriction solution.

4.2 The Interpretation-Restriction Solution Revisited

Let us take a step back and consider what types we would like to use for the translations of ordinary (e.g., *woman*, *fish*) and extraordinary (e.g., *temperature*, *price*) common nouns, considered separately. At each instant of time there should be a set of entities that are women (using this as an example), and the set of women should be able to vary from instant to instant. Ergo, type (s, (e, t)) is natural for translations of ordinary common nouns. Each temperature should be a function from instants to entities, and whether such a function should be a temperature should *not* vary from instant to instant. As the existence of more than one temperature should be possible, translations of extraordinary common nouns should be a characteristic function on the set of functions from instants to entities, that is, of type ((s, e), t).

As we would like (taking a step forward again) the types of interpretations of translations of common nouns to be the same (making less extensive modifications to Montague Grammar than Schwager proposes), we should "generalize to the worst case", that is, take the simplest type that can be "reduced" to (s, (e, t)) and to ((s, e), t) by "neutralizing" parts of it. That type is (s, ((s, e), t)), which is exactly the type assigned by Montague. He also took care of the neutralization of the inner index dependence for ordinary common nouns, by **MP2**. To obtain the desired reduction for extraordinary common nouns, the outer index dependence must be neutralized. And that is the effect of **MP3**. Thus we are back at the interpretation-restriction solution.

And this is indeed a good solution. The critique that Lasersohn mounts against it does not stand up to close inspection, being based entirely on the claim that **MP3** must be extended to ordinary common nouns to ensure the validity of the translation of arguments like this one (repeated from above):

> Necessarily, the woman is the fish The woman rises The fish rises

That claim is not correct. The validity of this argument is already ensured by **MP2**. The short explanation is this: The formal invalidity of the Gupta syllogism is a result of the double index dependency. Neutralizing either one of the inner and the outer index dependencies yields validity. The following is a more rigorous proof: The translation of the argument is

necessarily, the woman is the fish $\mapsto \quad \forall t (\exists y (\forall x (\mathbf{woman}_t(x) \leftrightarrow x = y) \land \exists v (\forall z (\mathbf{fish}_t(z) \leftrightarrow z = v) \land y_t = v_t)))$ the woman rises $\mapsto \quad \exists y (\forall x (\mathbf{woman}_t(x) \leftrightarrow x = y) \land \mathbf{rise}_t(y))$ the fish rises $\mapsto \quad \exists y (\forall x (\mathbf{fish}_t(x) \leftrightarrow x = y) \land \mathbf{rise}_t(y))$

Let a model satisfying **MP2** be given together with an instant of time in this model, such that the premises are true at that instant. From the first premise this follows: There is a unique function y from the set of instants to the set of entities such that (abusing notation in a harmless way) woman_t(y) is true, and a unique function vfrom the set of instants to the set of entities such that fish_t(v) is true, and y and vhave the same value at the instant under consideration. By **MP2**, y and v are constant functions. It follows that y = v. Further, it follows from the second premise that rise_t(y) is true at the given instant, so rise_t(v) is as well. Ergo, the conclusion is true.

Actually, when **MP3** is added for extraordinary common nouns, Montague Grammar correctly predicts the validity or lack thereof of all the arguments mentioned in this paper: the temperature argument (page 1) is formally invalid, and the Gupta syllogism (page 2), the "mermaid argument" (page 4), and the "a-every argument" (page 7) are all formally valid.

It should also be noted that accepting this solution does not prevent us from treating temperature and price in the functional way proposed by Schwager. Let me very briefly outline one way to do it (a proper treatment must be left to another occasion). We could add a constant of that denotes something that, when applied first to something of the type of terms and then to something of the type of common nouns, returns something of the type of common nouns. Then we can interpret, for example, *temperature of Frankfurt*. If $of_t(Frankfurt_t)(temp_t)$ denotes the characteristic function of a singleton, then *The temperature of Frankfurt rises* could come out true. This way of treating *of* would not restrict it to functional use, but could also be used relationally: as in, for example, the case of *a consul of Rome*.

5 Conclusion

The conclusion is that Lasersohn (2005) is wrong: the temperature paradox does not constitute evidence for a presuppositional analysis of definite descriptions. It can be handled perfectly well using Russell's analysis. An advocate of that analysis can simply take the significance of the Gupta Syllogism to be that it shows that Montague (1973) "missed" a single meaning postulate when he laid out his system.

Let me emphasize that this conclusion is merely negative, that is, I am not rejecting the solution suggested by Lasersohn and developed by Schwager. (It is more complicated, and that is a point against it, but it is a weak point.)

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