

Review of
“Plural Logic”
by Alex Oliver and Timothy Smiley

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Alex Oliver and Timothy Smiley have written a very interesting book. It is primarily concerned with the fact that the previous sentence is not equivalent to ‘Alex Oliver has written a very interesting book and Timothy Smiley has written a very interesting book’ and similar logical phenomena related to talking about multiple things at once rather than just *one* thing or *a* set of things. They argue that a plural (and free) logic should replace what they refer to as ‘singular logic’.

The book achieves a lot. It gives one a healthy appreciation of just how pervasive and varied the phenomenon of plurality is in natural language, and it forces one to take plural logic seriously. It is extremely well-informed on the history of its own subject, and critiques it with authority. It is well-written, well-argued and well-researched. Formalism has been used when and only when it is helpful and much less than one would expect in a book with ‘logic’ in its title. A deep knowledge of the subject is not presupposed.

Oliver and Smiley aim high. They do not merely attempt to get us to appreciate the particular virtues of plural logic; they want to convince us that, when it comes to choosing ourselves a logic, plural is the only way to go. We ultimately remained unconvinced of this stronger claim, but we commend Oliver and Smiley for pursuing it, as their attempt yields many interesting arguments.

Setting aside the final chapter and the postscript, *Plural Logic* can be divided into three parts. Going from back to front, in the last of these, comprising chapters 11 through 13, a formal plural logic is developed in stages. After giving their preferred version of singular logic, they supplement it first with plural variables, a plural exhaustive description operator and an inclusion predicate, yielding a still axiomatisable system which they call ‘mid-plural logic’. They then supplement this system with a plural quantifier and a plural definite description operator, yielding a fully functioning plural logic which is no longer axiomatisable. They compare the syntactic and semantic features of these systems, emphasising the expressive power of full plural logic, which they go on to illustrate in chapter 14 (discussed below).

In the middle part, chapters 5–10, Oliver and Smiley prepare the ground for their preferred approach to plural logic by considering various manifestations of plurality in natural language and mathematical practice. They look at different types of plural denoting terms, at plural predication (collective and distributive), at plural definite description, and at multivalued functions. In all of these cases they argue that plural language is both intelligible and philosophically respectable when taken at face value, and more difficult to dispense with than might at first be thought. Therefore, they argue, it is no mark in favour of a formal language that it is singular.

As a representative example of the type of arguments employed, we can mention one in chapter 9 against the now standard view that functions are relations, namely those relations that assign just one value to each value of the variable. The argument is that functions and relations belong to different grammatical categories: ‘Completing a function sign with argument terms produce a term, doing the same to a predicate produces a sentence’ (p. 146). Oliver and Smiley believe that such a difference in natural language has to be respected by formalizations and are much less inclined to accept an alleged distinction between ‘surface structure’ and ‘deep structure’ than your average logician. Having established the distinction between functions and relations as a difference of type, they can avoid the conflation of multivalued functions with relations, allowing, e.g., $\sqrt{\quad}$ to be interpreted as a function that maps any one number (except 0) to two values. $\sqrt{2}$ denotes, unambiguously, two numbers the same way that ‘Schiller and Goethe’ unambiguously denotes two men (p. 148).

The positive work in the middle part is prefaced by detailed critique of the opposition, first from an historical and chronological angle and with focus on Mill, Frege, Leśniewski, and Russell, and then systematically. They identify two main strategies for the singularist, each with a host of different ways of filling out the details. The first is to ‘change the subject’ and interpret talk of several objects as really being about just one thing, namely the set or collection or mereological sum of those objects. Oliver and Smiley argue that such a reinterpretation is bound to be a *mis*interpretation of natural language and that we should instead take such talk at face value as being about *some things*. The second is predicative analysis: a sentence with the surface structure $P(a)$ where a is a plural subject and P is a first-order predicate is interpreted as really being of the form $P\uparrow(a\uparrow)$ where $a\uparrow$ is a first-order predicate satisfied by exactly the objects (seemingly) denoted by a and $P\uparrow$ is a second-order version of P .

As a bonus, a final chapter is included in which Oliver and Smiley put their full plural logic to work, using it as a metalanguage for stating their preferred system of set theory, which, as it happens, is a system with no empty set and no singletons. They motivate these choices at length, and their arguments for them are interesting quite independently of the topic of

plural logic. Finally, a postscript lists, and very briefly discusses, a number of subjects for further work.

The book is something of a tour de force of linguistic analysis; Oliver and Smiley believe that the prevailing singularist orthodoxy is at least in part the result of an unbalanced diet of examples, and they are keen to correct this by showing us just how pervasive a feature of language and reasoning plurality is. When taken simply as a demonstration of what philosophical logicians of the past have missed and ignored, this is certainly a success. But Oliver and Smiley are not merely trying to show us that singularists will have a tough time adequately interpreting the things we say in natural language; they clearly believe that the pervasiveness of plurals in natural language gives us a general reason to prefer a logic which incorporates them, no matter what our purposes. It is therefore unfortunate that they never state any methodological principles. One would like to know exactly what they take the evidential role of natural language to be vis-à-vis the choice of a formal logic. When, in general, do we want a formal logic to preserve a given feature of natural language? The answer clearly cannot be ‘always’, for then we would not have any reason to abandon natural language for a formal language in the first place. We want something more selective; but what?

Even though Oliver and Smiley state no methodological or metaphilosophical principles, some such principles nevertheless shine through. First off, they seem to assume a form of logical monism: there is a single logic which is the preferable logic whatever our purposes. Such a view has been challenged frequently, and so deserves some motivation. The closest Oliver and Smiley come is stating that they believe that logic ought to be topic-neutral. A topic-neutral logic is presumably an all-purpose logic, and so they reject forms of logical pluralism which make the best choice of logic a purpose-relative matter. But since they do not attempt to argue for the claim that a good logic ought to be an all-purpose logic, this only takes us so far.

Secondly, they clearly take the analysis of natural language to be a major source of evidence when it comes to motivating a choice of logic. But it would have been nice if they had stated what they take the precise connection between natural language and logic to be. This is particularly urgent because although they devote a great deal of space to the analysis of natural language, they almost wholly dedicate themselves to one such language: English. Perhaps this is perfectly justifiable; perhaps the study of English gives us evidence enough. But one would like to know why.

There are various different routes by which natural language and logic might be connected. One might take it to be a desideratum that any argument that can be given in natural language can be reconstructed in one’s formal logic; if so, the formal logic should in principle be able to express anything one can express in natural language. If Oliver and Smiley show us that a singular

logic cannot, in principle, capture everything that one can say in a natural language with plurals, then this would be a reason for going plural. In their bolder moments, Oliver and Smiley do seem to take themselves to show this, though we reserve judgement on whether they succeed; though they do indicate a number of real challenges facing the ‘singularisation’ approach to interpreting plural talk, we are not quite convinced that they show that these challenges cannot in principle be overcome.

(In the postscript, Oliver and Smiley give examples of expressions that singularise pluralities in English: ‘a pair’, ‘a dozen’, ‘a hundred’, ‘a majority’ (p. 273). In these cases they deny the force of the evidence from surface structure and claim that these expressions are ‘not what they seem’ and *really* denote several things rather than one. We agree that it is reasonable not to take the syntax as ontologically committing here. However, taking that stance seems to spell problems for the authors. First of all, to explain away what natural language seems to tell us is to go against the methodology they adhere to in the rest of the book. Further, we think that the most plausible way to resolve that methodological contradiction is to do it in favour of this exception and relax the uncompromising attitude against singularisation that dominates the rest of the book. Singularisation seems to be an inherent and important part of natural language itself, rather than just an (objectionable) modeling technique of logicians. The usefulness is demonstrated by the unclarities created by the use of *superpluralities*. We don’t have to think up examples ourselves, but can find them in the book (p. 275–277): On the most natural reading of ‘Gilbert is one of the creators of great comic opera’ the sentence is true, but Oliver and Smiley take it to be false because they interpret ‘the creators of great comic opera’ to be a plurality of pluralities that includes Gilbert and Sullivan, but not Gilbert on his own since he didn’t write one by himself. Here superpluralities fail to get an unambiguous point across, and singularisation is superior: collect Gilbert and Sullivan into *one* team and express the intended meaning with ‘Gilbert is a member of one of the teams that created great comic opera’. The same point applies to Oliver and Smiley’s other examples; let us mention just one more. In a scenario where both some English boys together and some French boys together have solved a puzzle, it is far from obvious that Oliver and Smiley’s ‘the boys who solved the puzzle’ is supposed to denote those boys in the intended structured way where those who solved it together are grouped together, so the singularising ‘the groups of boys that solved the puzzle’ does a better job. Finally, if singularisation is both useful, legitimate, and ontologically innocent in natural language as it is, then it might also be useful, legitimate, and ontologically innocent to invent new ways to employ it more extensively in artificial languages, which is what singularising logicians have done.)

Of course, one might doubt that it is unreservedly a good thing for a formal language to be able to state anything natural language can. Many take

the design of formal languages to be a matter of reconciling sometimes conflicting desiderata; an increase in expressive power might yield undesirable consequences such as inconsistency or, in the case of plural logic, unaxiomatisability. If one does take there to be good reason to sometimes sacrifice expressive power in favour of something else, one would have to live with the possibility that there are things we cannot reason about using formal languages. But one might be fine with that. Oliver and Smiley seem to value expressive power very highly. That is their prerogative; but it would have been better if they had done more to convince us that we should too.

Concerning expressive power, we also feel a need to criticize the authors' use of the paradoxes of set theory in their argumentation. They point out (p. 41) that the set theoretic singularist, who interprets 'a is one of **b**', where '**b**' is a plural term, as 'a is an element of ϕ ', where ϕ is the set of **bs**, bumps into a problem when trying to talk about all sets. She ends up being committed to the paradoxical '{Whitehead,Russell} is an element of the set of all sets that are not elements of themselves' when trying to express that {Whitehead,Russell} is one of the sets (plural) that are not elements of themselves, which Oliver and Smiley take to be a true proposition. However, in order not to fall victim to Cantor's Paradox themselves, they deny thing-hood to pluralities and to functions, pronouncing the latter *sui generis* entities (p. 214). But they make essential use of functions in the meta-language when specifying semantics and there are more of those functions than there are things. How is it possible to express propositions about them? Of course, Oliver and Smiley could consistently claim that it is legitimate to use a more extensive meta-language when needed (if we set aside their demand for complete topic-neutrality). But so could the singularist: she could turn to proper classes when needing to talk about all sets. Neither party has achieved unrestricted generality.

One consideration the authors appeal to which might reveal some of their methodological principles is uniform translatability. If in English we find that we can reason about an individual and about a plurality in pretty much analogous ways, simply pluralising expressions where necessary and leaving everything else unchanged, then our logic should preserve that analogy. We are not necessarily unsympathetic to this sort of consideration, but it nevertheless remains unclear what justifies it. It could simply be a principle of conservativeness; if there is no positive reason to do otherwise, let your formal language preserve the structure of natural language, as this makes it easier to interpret the latter using the former. If that is to be the thought, then Oliver and Smiley can be interpreted as arguing that all purported reasons for breaking the analogy between singular and plural talk in one's formal system are spurious, and so conservativeness wins the day. But some more powerful principle could be at work. Perhaps the singular-plural analogy in English can be taken as an indication that singular and plural *thought* works in essentially the same way, and our formal logic should

preserve this if at all possible.

Although the book covers a lot of ground, the discussion remains solidly within the confines of the philosophy of logic. One thing we would have enjoyed is a discussion, even just a tentative one, of how the phenomenon of plurality crops up in other areas of philosophy, and how plural logic might be of use there.

For example, a philosopher of mind might be interested in and puzzled by plural intentionality. You are able, somehow, to direct your thoughts to a plurality of things; you can think about the unmarked essays on your table, and how very many they are. What are you doing, when thinking of a plurality? One story is that you engage in many intentional acts simultaneously, each aimed at one essay. But that seems wrong. Another story is that you engage in a single act aimed at a single object, the collection of essays. But that is not obviously the case. What you seem to be doing on the face of it is referring, in one act, to multiple things, as multiple things, but ‘together’. That seems right: but what does the phrase ‘together’ express here? Intentionality is often explained with the metaphor of ‘directedness’: a thought is about something iff it is directed at that thing. But that metaphor seems inadequate in the case of plural intentionality. Can one thought really be aimed in multiple directions simultaneously? And if not, should we take from that that the metaphor is faulty, or that plural intentionality is somehow philosophically problematic? Oliver and Smiley do include a discussion of plural denotation (Ch. 6), which takes them close to such issues, but they do not seem interested in exploring them.

Another example is from metaphysics. Those who work in fundamental metaphysics often wonder what sorts of phenomena can be found at the ‘bottom level’ of reality. Are modality and temporal ordering irreducible features of reality, or do they arise from more fundamental features? Are there fundamental relations, or are the fundamental properties all one-place properties? Along those same lines one might wonder about plurality. Is it conceivable that at the fundamental level, reality is fully singular? That is, might fundamental reality consist in individual objects individually having properties, all plural facts somehow arising therefrom? Or are there fundamental plural facts? If the former, then one might hold that, though a plural logic might be useful to us, or even indispensable given the way we think, an adequate account of the world might in principle be given in a singular logic. If the latter, plural logic is indispensable in a much more profound sense.

In conclusion, *Plural Logic* is a notable achievement. Though it might fall short of its lofty goal of showing us we simply cannot do without plural logic, it certainly manages, in a very thorough way, to show us just how much we would be doing without.